A STUDY ON MANUFACTURING OF THE FORM ROLLING DIE OF TOOTH SURFACE OF HOURGLASS WORM --First Report--

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Abstract In the field of automobile parts, a large amount of cylindrical worm gear are used. For example, they are power windows, wipers, seats and steerings. The big concern in this field is how to go up the efficiency and go down the size of the worm gear set for the environmental and cost problems. Recently, the hourglass worm gear has been observed with interest by its higher efficiency, higher load carrying capacity and longer life compared with former ones in general industry fields. But the size of the gear set in automobile field must be smaller than it is in the other industry field and at the same time manufacturing way of the gear sets must be fitted to mass production.

So, in this paper I propose the form rolling die for forming the teeth surface of hourglass worm so as to solve the problem of its mass production. In the 1st report I show the calculation of teeth figure of rolling die and worm formed with this pair of rolling dies. In the 2nd report, I will show the result of the manufacturing test of the form rolling die and worm.

key words: Form Rolling Die, Hourglass Worm Gearing, Automobile-Related Hourglass Worm

1. Introduction

Cylindrical worm gearing is used in many automobile-related machineries. For example, it is used in wipers, window systems, sheets and electric power steering (EPS) systems. The miniaturization and high efficiency of these machineries are particularly important for realizing cost reduction and energy savings for automobiles.

In recent years, it has been clarified that hourglass worm gearings that are finished by grinding and have a hardened tooth surface have high efficiency, large load carrying capacity and long life compared with those of cylindrical worm gearings; as a consequence, cylindrical worm gearings have been gradually replaced by hourglass worm gearings for general industrial use.

The worm gearings used in the automobile industry have a smaller center distance than those applied for general industrial use. In addition, the worm gearings used in the automobile industry are required to be mass-produced on a much larger scale than those applied for general industrial use in order to satisfy the demand. The mass-production of cylindrical worm gearings has already been realized, however, although the performance of the hourglass worm gearings may be superior, their practicability is limited before mass production of the hourglass worm gearings is realized.

In this paper, a method of resolving the mass production issue, by adopting form rolling of the tooth surface of the hourglass worm(Note 1) is proposed. In this first report, the theoretical analytical results are discussed, and in the second report, the actual experimental results of form rolling will be reported.

(**Note 1**) Since the tooth surface of an hourglass worm is a screw surface with a nonconstant lead, in order to distinguish the form rolling of the tooth surface of an hourglass worm from that of the conventional screw surface with a constant lead, the former is called form rolling of the tooth surface, in this paper.

2. Theoretical Analysis

2.1 Basic concept

There are two types of die, roller die and flat die, which are used in the form rolling of the tooth surface of an hourglass worm. In this study, since the tooth surface of a die is analyzed using the theory of skew gears,[1] only the roller die is analyzed. If a roller die is used to fabricate cylindrical worm screws, the roller die has a cylindrical external shape; however, if it is used for the fabrication of an hourglass worm, a beer barrel shape is adopted as an external shape. The form-rolled tooth surface of an hourglass-worm (W surface, hereafter) and the tooth surface of a beer-barrel-shaped die used in form rolling (D surface, hereafter) mesh with each other while both axes of rotation are parallel. In

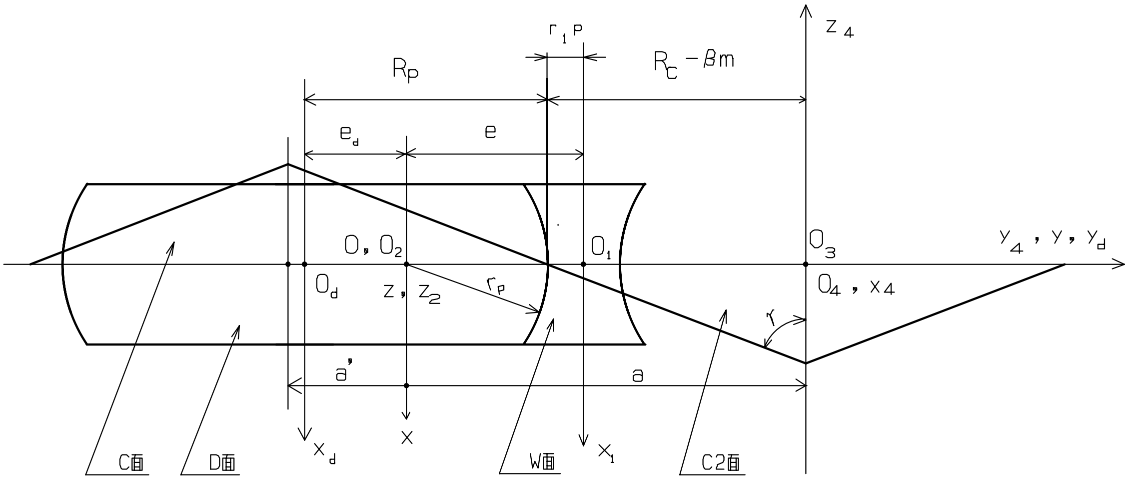

Fig. 1 Positional relationship of the die processing axis used to form roll hourglass worm

other words, the W surface is rolled and formed under the above condition.

The W surface can be produced using either a conical tool surface (C tool surface, hereafter)[(2)] or reverse-conical tool surface (MC tool surface, hereafter).[(3)] In this study, since a direct generating method is adopted, the center of rotation of these tools corresponds with that of the wheel. However, there is a slight difference between the worm tooth surface by C tool and by MC tool. Similarly, if the D surface is hypothetically generated from inside (from the material side) by the C tool surface which is used to generate the W surface, the W and D surfaces come into contact with each other .

In actuality, since it is not possible to generate a D surface from inside (from the material side), the D surface is generated using a second conical tool surface (C2 tool surface, hereafter), which is the second conical tool surface that is symmetrical to the C tool surface with respect to the pitch point . As shown in Fig. 1, when the wheel axis is placed on the left-hand side with respect to the worm axis, the conical tool axis and die axis also exist on the left-hand side; however, the MC tool axis and the C2 tool axis exist on the right-hand side. The centers of rotation of C and C2 tools correspond to that of the wheel as explained before. Since the direct generating method is adopted, the angular velocities of C and C2 tools also correspond to that of the wheel. The D surface generated by the C2 tool surface is slightly different from the D surface generated by the C tool surface. This relationship resembles the relationship between the C tool surface, which generates the W surface, and the MC tool surface.

Accordingly, the wheel tooth surface, which meshes with the form-rolled worm, should be generated using a tool having the same shape as that of the form-rolled hourglass worm, similar to the case of generating an hourglass worm gearing using the MC tool surface.

2.2 Setting of coordinates and definition of symbols

In Figs. 1 and 3,

O-xyz: stationary coordinate system in which the z-axis (perpendicular to this paper) corresponds to the wheel axis and the y-axis is a common perpendicular line.

O_4-$x_4y_4z_4$: C2-tool-axis fixed coordinate system in which the conical principal axis is the z_4 axis. (In Fig. 1, the z_4 axis intersects the paper at point O_4.)

O_2-$x_2y_2z_2$: wheel-axis fixed coordinate system in which the z_2 axis corresponds to the wheel axis, z.

O_1-$x_1y_1z_1$: worm-axis fixed coordinate system in which the x_1 axis corresponds to the worm axis.

O_d-$x_dy_dz_d$: die-axis fixed coordinate system in which the x_d axis corresponds to the die axis.

The angular velocities of the worm, wheel and die around O_1, O_2 and O_d axes are ω_1, ω_2 and ω_d, respectively. The distances between the two axes are expressed as $OO_1 = e$, $OO_d = e_d$, $OO_3 = a$ and $O_3O_4 = b$.

The following is a summary of the other symbols.

r_p: radius of wheel reference circle

r_{1p}: radius of reference circle of worm throat part

r_{1f}: radius of tooth bottom of worm throat part

R_p: radius of die pitch circle

R_f: radius of tooth bottom of die

R_c: radius of conical tool

γ : half angle of conical tool

δ : inclination angle of tool

ϕ : angle of rotation of wheel axis = angle of rotation of tool axis

w_c: width of the tool tip

m: axial module

i: angular velocity ratio of worm to wheel = angular velocity ratio of worm to tool = ω_1 / ω_2

k: angular velocity ratio of worm to die = ω_1 / ω_d

j: angular velocity ratio of wheel to die = angular velocity ratio of tool to die = ω_d / ω_2

2.3 Equations for tool

As explained in Section 2.2, the position of the C2 tool is expressed as $\boldsymbol{X_4}$(u, v) by the tool-axis fixed coordinate system O_4, and its normal vector is expressed as $\boldsymbol{n_4}$(u, v) (Fig. 2). Since the C2 tool rotates around the center of the wheel axis to generate the die tooth surface, when the above coordinates are transformed into O_2 coordinates (Fig. 3),

$$\boldsymbol{X_2} = \boldsymbol{A_3}\boldsymbol{X_4} + \boldsymbol{B} \tag{1}$$

and

$$\boldsymbol{n_2} = \boldsymbol{A_3}\boldsymbol{n_4} \tag{2}$$

are obtained. In stationary coordinates

$$\boldsymbol{X} = \boldsymbol{A_2}\boldsymbol{X_2} \tag{3}$$

and

$$\boldsymbol{n} = \boldsymbol{A_2}\boldsymbol{n_2} \tag{4}$$

Here, $\boldsymbol{X_4}$, $\boldsymbol{n_4}$, $\boldsymbol{A_2}$, $\boldsymbol{A_3}$ and $\boldsymbol{B}$ are represented as follows.

$$\boldsymbol{X}_4 = v\begin{pmatrix} \sin\gamma\cos u \\ \sin\gamma\sin u \\ \cos\gamma \end{pmatrix} \quad \boldsymbol{A}_2 = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{A}_3 = \begin{pmatrix} -\sin\delta & 0 & -\cos\delta \\ 0 & 1 & 0 \\ \cos\delta & 0 & -\sin\delta \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} b\cos\delta \\ a \\ b\sin\delta \end{pmatrix}$$

$$\boldsymbol{n}_4 = \begin{pmatrix} \cos\gamma\cos u \\ \cos\gamma\sin u \\ -\sin\gamma \end{pmatrix} \tag{5}$$

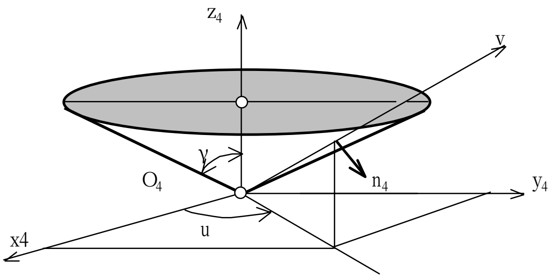

Fig. 2 Coordinates for conical tool

2.4 Equations used to obtain tooth surface of die

The die material is attached to the x_d axis of the O_d coordinate system, and rotates around the x_d axis at ω_d. Against this material, the C2 tool rotating around O_2 axis at ω_2 contacts the die to generate die tooth surface, X_d. Here, the C2 tool and the die tooth surface satisfy the contact condition[4] proposed by Sakai. If the die tooth surface is represented by stationary coordinates (Fig. 3),

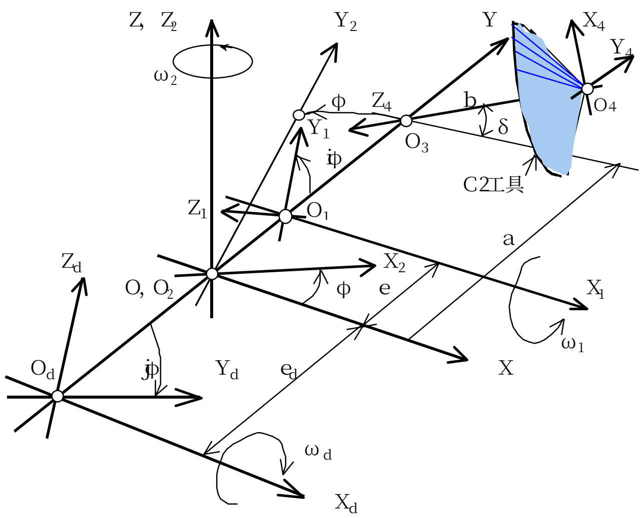

Fig. 3 Positional relationship of C2-tool-axis fixed coordinates and die-axis fixed coordinates

$$\boldsymbol{X} = \boldsymbol{A_d}\boldsymbol{X_d} + \boldsymbol{C_d} \tag{6}$$

$$\boldsymbol{n} = \boldsymbol{A_d}\boldsymbol{n_d} \tag{7}$$

and

$$\boldsymbol{A}_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos j\varphi & -\sin j\varphi \\ 0 & \sin j\varphi & \cos j\varphi \end{pmatrix} \quad \boldsymbol{C_d} = \begin{pmatrix} 0 \\ e_d \\ 0 \end{pmatrix} \tag{8}$$

Based on the contact conditions between a tool and a die in stationary coordinates, the contact lines between the tool and the die, as well as the tooth surface of the die are calculated. When the tool velocity in the stationary coordinate, X, is represented as V_2, die velocity as V_d and the relative velocity of the tool and die as W,

$$\boldsymbol{W} = \boldsymbol{V_2} - \boldsymbol{V_d} = \boldsymbol{\omega_2} \times \boldsymbol{X} - \boldsymbol{\omega_d} \times (\boldsymbol{X} - \boldsymbol{C_d}) \quad (9)$$

The equation for contact conditions between the two is

$$\boldsymbol{n} \cdot \boldsymbol{W} = 0 \quad (10)$$

Since $\omega_d/\omega_2 = j$ holds, the equation for contact conditions is then represented as

$$-y \cdot n_x + (x + j \cdot z) \cdot n_y - j \cdot (y - e_d) \cdot n_z = 0 \quad (11)$$

using Eqs. (9) and (10). By substituting Eqs. (1)-(5) into Eq. (11), x, y and z are calculated.

Assuming

$$A1 = \cos\varphi(\sin\delta\sin\gamma\cos u + \cos\delta\cos\gamma) + \sin\varphi\sin\gamma\sin u$$
$$B1 = \sin\varphi(\sin\delta\sin\gamma\cos u + \cos\delta\cos\gamma) - \cos\varphi\sin\gamma\sin u$$
$$C1 = \cos\delta\sin\gamma\cos u - \sin\delta\cos\gamma$$
$$A2 = b \cdot \cos\varphi\cos\delta - a \cdot \sin\varphi$$
$$B2 = b \cdot \sin\varphi\cos\delta + a \cdot \cos\varphi$$
$$C2 = b \cdot \sin\delta \quad (12)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -v \cdot A_1 + A_2 \\ -v \cdot B_1 + B_2 \\ v \cdot C_1 + C_2 \end{pmatrix} \quad (13)$$

holds. Based on Eqs. (11)-(13),

$$v = \frac{B_2 \cdot n_x - (A_2 + j \cdot C_2) \cdot n_y + j \cdot (B_2 - e_d) \cdot n_z}{B_2 \cdot n_x - (A_2 + j \cdot C_2) \cdot n_y + j \cdot (B_2 - e_d) \cdot n_z} \quad (14)$$

By substituting Eq. (14) into Eq. (13), a contact line in stationary coordinate is obtained. In addition, by reverse-converting Eq. (6), the contact line on die-axis fixed coordinates is calculated as follows.

$$\boldsymbol{X_d} = \boldsymbol{A_d}^{-1} (\boldsymbol{X} - \boldsymbol{C_d}) \quad (15)$$

$$\boldsymbol{n_d} = \boldsymbol{A_d}^{-1} \boldsymbol{n} \quad (16)$$

$$\boldsymbol{A}_d^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos j\varphi & \sin j\varphi \\ 0 & -\sin j\varphi & \cos j\varphi \end{pmatrix} \quad (17)$$

Assuming $z_d=0$, the cross-sectional tooth surface of the die axis is obtained as

$$x_d = x_d(\phi), \quad y_d = y_d(\phi) \quad (18)$$

2.5 Shape of die

In this section, the shape requirements of the die used to form roll the tooth surface of the hourglass worm are discussed. Here, mechanical issues are focused on, and only meshing performance between the die tooth surface and the form-rolled worm tooth surface is discussed, while issues related to plasticity are excluded. The worm is hourglass-shaped and the die is beer-barrel-shaped. The worm and the die mesh with each other and rotate around the x_1 and x_d axes, respectively, which are parallel. When the worm and the die are focused on, each has a pitch point and a pitch circle. Since the angular velocity ratio of worm to die at the pitch point is $\omega_1/\omega_d = R_p/r_{1p} = k$,

$$R_p = k \cdot r_{1p} \quad (19)$$

holds.

Table 1 the relationship between Hourglass worm and die

Items	Hourglass worm	Form-rolling die
Angular velocity ratio	$i=\omega_1/\omega_2$	$j=\omega_d/\omega_2$ $=i/k$
Center distance	e	$e_d=(1+k)r_{1p}-e$
Helix direction of thread	Right hand(Left)	Left hand(Right)
"a" dimension	$e-R_c-r_{1f}=$ $(e-r_{1p})-(Rc-\beta m)$	$R_c+R_f-e_d=$ $(e-r_{1p})+(Rc-\beta m)$

Table 1 summarizes the relationship between various parameters of the hourglass worm and the die used for form rolling during generation of the tooth surface. The "a" dimension is calculated from Fig. 1.

2.6 Tooth figure of rolled tooth surface

The die tooth surface is represented by Eqs. (15) and (19), which were used to calculate the screw surface of the form-rolled hourglass screw. The angle of rotation of the wheel axis used to generate the die tooth surface is represented by ϕ. While, θ represents the angle of rotation of the wheel during form rolling of the tooth surface of the hourglass worm using the die.

When the die tooth surface expressed by Eqs. (15) and (16) is represented by a stationary coordinate system,

$$\boldsymbol{X_\theta} = \boldsymbol{A_{d\theta}} \boldsymbol{X_d} + \boldsymbol{C_d} = \boldsymbol{A_{d\theta}} \boldsymbol{A_d}^{-1} (\boldsymbol{X} - \boldsymbol{C_d}) + \boldsymbol{C_d} \quad (20)$$

$$\boldsymbol{n_\theta} = \boldsymbol{A_{d\theta}} \boldsymbol{n_d} = \boldsymbol{A_{d\theta}} \boldsymbol{A_d}^{-1} \boldsymbol{n} \quad (21)$$

The terms of the transformation matrix of each of the coordinates are

$$A_{d\theta}=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{pmatrix} \tag{22}$$

$$A_{d\theta}A_d^{-1}=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\dot{\varphi} & \sin\dot{\varphi}\\ 0 & -\sin\dot{\varphi} & \cos\dot{\varphi}\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta-\varphi) & -\sin(\theta-\varphi)\\ 0 & \sin(\theta-\varphi) & \cos(\theta-\varphi)\end{pmatrix} \tag{23}$$

Using Eqs. (20) and (21), the die tooth surface is represented by a stationary coordinate system. Assuming worm tooth surface is represented as X_1 by a worm-axis fixed coordinate system, its representation by stationary coordinates becomes

$$X_\theta=A_1X_1+C_1 \tag{24}$$

$$A_1=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\dot{\varphi} & -\sin\dot{\varphi}\\ 0 & \sin\dot{\varphi} & \cos\dot{\varphi}\end{pmatrix} \quad C_1=\begin{pmatrix}0\\ e\\ 0\end{pmatrix} \tag{25}$$

The relative velocity of the die and the worm, W_d, is represented as

$$W_d=V_1-V_d$$

$$=\omega_1\times(X_\theta-C_1)-\omega_d\times(X_\theta-C_d) \tag{26}$$

using Eqs. (20) and (24).

Based on Eqs. (26) and (21), the equation for the contact condition is

$$n_\theta\cdot W_d=0 \tag{27}$$

Since $\omega_1/\omega_d=k$ holds, the equation for the contact condition is rewritten as

$$M=(e_d-k\cdot e)/(1-k) \tag{28}$$

using Eqs. (26) and (27).

Here, M is a constant and

$$z_\theta\cdot n_{y\theta}+(M-y_\theta)\cdot n_{z\theta}=0 \tag{29}$$

By substituting Eqs. (12) and (20)-(23) into Eq. (29), $(x_\theta, y_\theta, z_\theta)$ is obtained. Then,

$$v=\{n_z\cdot(B_2-e_d)-C_2\cdot n_y-(M-e_d)\cdot n_{z\theta}\}/(C_1\cdot n_y+B_1\cdot n_z) \tag{30}$$

v is given by Eq. (14) and then u, which satisfies the Eq. (30), is obtained.

$$u=u(\phi,\ \theta) \tag{31}$$

By substituting u into Eq. (20), the contact line between the die tooth surface and tooth surface of the form-rolled worm in stationary coordinates is obtained as a function of φ and θ. In addition, using Eq. (24),

$$X_1=A_1^{-1}(X-C_1) \tag{32}$$

$$A_{1\theta}^{-1}=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\dot{\theta} & \sin\dot{\theta}\\ 0 & -\sin\dot{\theta} & \cos\dot{\theta}\end{pmatrix} \tag{33}$$

The X_1 in Eq. (32) represents the tooth surface of the form-rolled worm. Assuming $z_1=0$, the cross-sectional tooth figure of the worm axis is obtained as

$$x_1=x_1(\theta),\quad y_1=y_1(\theta) \tag{34}$$

3. Conclusion

The possibility of producing form-rolled hourglass worms has been increased based on the results of this analysis. In the second report, we will report on a design example of dies based on the analysis and comparison between the calculated and experimental results of the tooth surface of the worms which are form-rolled using the die.

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